

On self-complementarity relations of neutrino mixing

Xinyi Zhang^a, Bo-Qiang Ma^{a,b,*}

^a*School of Physics and State Key Laboratory of Nuclear Physics and Technology,
Peking University, Beijing 100871, China*

^b*Center for High Energy Physics, Peking University, Beijing 100871, China*

Abstract

With the latest results of a large mixing angle θ_{13} for neutrinos by the T2K, MINOS and Double Chooz experiments, we find that the self-complementarity (SC) relations agree with the data in some angle-phase parametrizations of the lepton mixing matrix. There are three kinds of self-complementarity relations: (1) $\vartheta_i + \vartheta_j = \vartheta_k = 45^\circ$; (2) $\vartheta_i + \vartheta_j = \vartheta_k$; (3) $\vartheta_i + \vartheta_j = 45^\circ$ (where i, j, k denote the mixing angles in the angle-phase parametrizations). We present a detailed study on the self-complementarity relations in nine different angle-phase parametrizations, and also examine the explicit expressions in reparametrization-invariant form, as well as their deviations from global fit. These self-complementarity relations may lead to new perspective on the mixing pattern of neutrinos.

Key words: neutrino, mixing matrix, mixing angle, self-complementarity (SC) relation

* Corresponding author.

Email address: mabq@pku.edu.cn (Bo-Qiang Ma).

One of the most interesting issues concerning neutrinos is the misalignment of the flavor eigenstates with the mass eigenstates, which is the cause of the oscillations as in the neutrino oscillation theory and is described by the mixing matrix phenomenologically [1]. Many experiments are set to get the parameters of the oscillations well determined so as to get insights into the intriguing nature of neutrinos [2,3,4].

Just like the Cabibbo-Kobayashi-Maskawa (CKM) matrix [5] describing the mixing of quarks, the misalignment of the flavor eigenstates with the mass eigenstates in the lepton sector can also be described by a mixing matrix which is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [6]. The PMNS matrix is defined as $U_{\text{PMNS}} = U_L^\dagger U_L^\nu$ and can be expressed generally as

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \quad (1)$$

Both the CKM and PMNS matrices can be parameterized by three rotation angles and a phase angle corresponding to CP violation as in the standard parametrization i.e. Chau-Keung (CK) parametrization [7]

$$\begin{aligned} U_{\text{CK}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\text{CK}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\text{CK}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CK}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CK}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CK}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CK}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CK}}} & c_{23}c_{13} \end{pmatrix}. \quad (2) \end{aligned}$$

Two additional phase angles are needed for the PMNS matrix if the neutrinos are of Majorana type. For neutrino mixing, the Majorana phase angles do not affect the absolute values of the elements of mixing matrix and are omitted in the above expression.

The angle-phase parametrizations refer to the methods that parametrize the mixing matrix with three rotation angles and one (Dirac) or three (Majorana) phase angles. Because a real and orthogonal matrix can always be decomposed as a product of three rotations of certain planes, there are options about how to arrange the orders of these three rotations. Of the twelve ways to do the

product, only nine are independent and the standard parametrization is one of the nine [8,9].

The experimental progresses have upgraded our understanding towards neutrino mixing in the past several decades. In the attempts of approximating the PMNS matrix to a constant matrix and proposing the corresponding theoretical framework on flavor symmetry, the bimaximal mixing (BM) [10]

$$U_{\text{BM}} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}, \quad (3)$$

and tribimaximal mixing (TB) [11]

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \quad (4)$$

have been selected as good description of the data. When working out the mixing angles in the standard parametrization, both of these two mixing patterns predict a vanishing mixing angle θ_{13} , which seems to be challenged by the indication of a relatively large θ_{13} in the recent T2K, MINOS and Double Chooz experiments [3]. Therefore new perspectives concerning the mixing pattern of neutrinos are needed to accommodate the new experimental results.

One interesting direction in the investigation of mixing matrices of quarks and leptons is trying to find some phenomenological relations among the mixing parameters, and then trying to find some theoretical backgrounds or frameworks to understand these relations. One example is the quark-lepton complementarity [12] between the mixing angles of quarks and leptons. Different parametrizations are equivalent to each other mathematically, while some phenomenological relations are parametrization-dependent. It is worthwhile to explicitly work out the relations in all the angle-phase parametrizations as Zheng did in Ref. [8] for the quark-lepton complementarity, and find some reparametrization-invariant expressions [13]. From the recent T2K result for a relatively large θ_{13} , there has been a proposal [14] of some self-complementarity relations among the mixing angles of the neutrino mixing matrix in the standard parametrization. The purpose of this work is to explore the phenomenological relations among the mixing angles for the PMNS mixing matrix, and then check their parametrization-dependence and -independence in the previously mentioned nine different angle-phase parametrizations, by confronting

with latest experimental results.

We start from a latest global fitting result of the PMNS mixing matrix [13,15],

$$|U_{\text{PMNS}}| = \begin{pmatrix} 0.824^{+0.011(+0.032)}_{-0.010(-0.032)} & 0.547^{+0.016(+0.047)}_{-0.014(-0.044)} & 0.145^{+0.022(+0.065)}_{-0.031(-0.113)} \\ 0.500^{+0.027(+0.076)}_{-0.021(-0.071)} & 0.582^{+0.050(+0.139)}_{-0.023(-0.069)} & 0.641^{+0.061(+0.168)}_{-0.023(-0.063)} \\ 0.267^{+0.044(+0.123)}_{-0.027(-0.088)} & 0.601^{+0.048(+0.133)}_{-0.022(-0.069)} & 0.754^{+0.052(+0.143)}_{-0.020(-0.054)} \end{pmatrix}, \quad (5)$$

which is obtained from a global fitting of neutrino mixing angles based on previous experimental data and T2K and MINOS experiments (1σ (3σ)) [15] together with an Ansatz of a null CP violating phase angle [13]. In our analysis, we calculate the mixing angles of the nine angle-phase parametrizations with matrix elements that are independent of the phase angle. For example, from the P1 parametrization, we have,

$$\sin \vartheta_{13} = |U_{e3}|, \quad \tan \vartheta_{12} = \frac{|U_{e2}|}{|U_{e1}|}, \quad \tan \vartheta_{23} = \frac{|U_{\mu 3}|}{|U_{\tau 3}|},$$

thus we get the corresponding values of the mixing angles. The results are listed in Table 1. Notice that there is no information on the CP violating phases of both Dirac and Majorana types for neutrinos from the experiments at present.

From Table 1, we can see that there are three kinds of self-complementarity relations (SC) [14] satisfied numerically in five angle-phase parametrizations. Explicit forms of the self-complementarity relations are underlined in the third column of the table and can be summarized as follows:

- (1) the first kind, which works in the P4 parametrization, suggests,

$$\vartheta_i + \vartheta_j = \vartheta_k = 45^\circ;$$

- (2) the second kind, which works in the P1 and P9 parametrizations, suggests,

$$\vartheta_i + \vartheta_j = \vartheta_k;$$

- (3) the third kind, which works in the P3 and P6 parametrizations, suggests,

$$\vartheta_i + \vartheta_j = 45^\circ.$$

In the above relations, i , j and k denotes different mixing angles. For the P1 parametrization, which corresponds to the standard parametrization with a slight difference in phase convention, we adopt ij to denote different mixing angles as in most literature. In fact, the first kind relation has been proposed by Xing in Ref. [16] from the viewpoint of adopting a democratic correction

Table 1

The angle-phase parametrizations and self-complementarity relations

parametrization	$\vartheta_1/\vartheta_2/\vartheta_3$	self-complementarity
P1: $U = R_{23}(\vartheta_{23})R_{13}(\vartheta_{13}, \phi)R_{12}(\vartheta_{12})$ $\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\phi} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\phi} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\phi} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\phi} & c_{23}c_{13} \end{pmatrix}$	$\vartheta_{12} = (33.58_{-0.496}^{+0.577})^\circ$ $\vartheta_{23} = (40.37_{-0.366}^{+0.505})^\circ$ $\vartheta_{13} = (8.34_{-1.802}^{+1.279})^\circ$	$\vartheta_{12} + \vartheta_{13} = (41.92_{-2.298}^{+1.856})^\circ$ $\underline{\vartheta_{12} + \vartheta_{13} \simeq \vartheta_{23}}$
P2: $U = R_{12}(\vartheta_3)R_{23}(\vartheta_2, \phi)R_{12}^{-1}(\vartheta_1)$ $\begin{pmatrix} s_1c_2s_3 + c_1c_3e^{-i\phi} & c_1c_2s_3 - s_1c_3e^{-i\phi} & s_2s_3 \\ s_1c_2c_3 - c_1s_3e^{-i\phi} & c_1c_2c_3 + s_1s_3e^{-i\phi} & s_2c_3 \\ -s_1s_2 & -c_1s_2 & c_2 \end{pmatrix}$	$\vartheta_1 = (23.95_{-2.824}^{+4.304})^\circ$ $\vartheta_2 = (41.06_{-1.658}^{+4.311})^\circ$ $\vartheta_3 = (12.75_{-3.892}^{+2.464})^\circ$	$\vartheta_1 + \vartheta_3 = (36.70_{-6.716}^{+6.768})^\circ$ $\underline{\vartheta_2 + \vartheta_3 \simeq \vartheta_1}$
P3: $U = R_{23}(\vartheta_2)R_{12}(\vartheta_1, \phi)R_{23}^{-1}(\vartheta_3)$ $\begin{pmatrix} c_1 & s_1c_3 & -s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{-i\phi} & -c_1c_2s_3 + s_2c_3e^{-i\phi} \\ s_1s_2 & -c_1s_2c_3 + c_2s_3e^{-i\phi} & c_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$\vartheta_1 = (34.51_{-0.739}^{+0.813})^\circ$ $\vartheta_2 = (28.10_{-3.441}^{+5.945})^\circ$ $\vartheta_3 = (14.85_{-5.251}^{+3.652})^\circ$	$\vartheta_2 + \vartheta_3 = (42.95_{-8.692}^{+9.597})^\circ$ $\underline{\vartheta_2 + \vartheta_3 \simeq 45^\circ}$
P4: $U = R_{23}(\vartheta_2)R_{12}(\vartheta_1, \phi)R_{31}^{-1}(\vartheta_3)$ $\begin{pmatrix} c_1c_3 & s_1 & -c_1s_3 \\ -s_1c_2c_3 + s_2s_3e^{-i\phi} & c_1c_2 & s_1c_2s_3 + s_2c_3e^{-i\phi} \\ s_1s_2c_3 + c_2s_3e^{-i\phi} & -c_1s_2 & -s_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$\vartheta_1 = (33.16_{-1.014}^{+1.159})^\circ$ $\vartheta_2 = (45.92_{-0.160}^{+0.338})^\circ$ $\vartheta_3 = (9.98_{-2.487}^{+1.755})^\circ$	$\vartheta_1 + \vartheta_3 = (43.14_{-3.501}^{+2.914})^\circ$ $\underline{\vartheta_1 + \vartheta_3 \simeq \vartheta_2 \simeq 45^\circ}$
P5: $U = R_{31}(\vartheta_3)R_{23}(\vartheta_2, \phi)R_{12}^{-1}(\vartheta_1)$ $\begin{pmatrix} -s_1s_2s_3 + c_1c_3e^{-i\phi} & -c_1s_2s_3 - s_1c_3e^{-i\phi} & c_2s_3 \\ s_1c_2 & c_1c_2 & s_2 \\ -s_1s_2c_3 - c_1s_3e^{-i\phi} & -c_1s_2c_3 + s_1s_3e^{-i\phi} & c_2c_3 \end{pmatrix}$	$\vartheta_1 = (40.67_{-0.324}^{+0.796})^\circ$ $\vartheta_2 = (39.87_{-1.864}^{+4.943})^\circ$ $\vartheta_3 = (10.89_{-2.907}^{+1.928})^\circ$	$\vartheta_2 + \vartheta_3 = (50.76_{-4.771}^{+6.871})^\circ$ $\underline{\vartheta_2 + \vartheta_3 \simeq \vartheta_1}$
P6: $U = R_{12}(\vartheta_1)R_{31}(\vartheta_3, \phi)R_{23}^{-1}(\vartheta_2)$ $\begin{pmatrix} c_1c_3 & c_1s_2s_3 + s_1c_2e^{-i\phi} & c_1c_2s_3 - s_1s_2e^{-i\phi} \\ -s_1c_3 & -s_1s_2s_3 + c_1c_2e^{-i\phi} & -s_1c_2s_3 - c_1s_2e^{-i\phi} \\ -s_3 & s_2c_3 & c_2c_3 \end{pmatrix}$	$\vartheta_1 = (31.25_{-0.960}^{+1.272})^\circ$ $\vartheta_2 = (38.56_{-0.484}^{+0.779})^\circ$ $\vartheta_3 = (15.49_{-1.625}^{+2.649})^\circ$	$\vartheta_1 + \vartheta_3 = (46.74_{-2.585}^{+3.921})^\circ$ $\underline{\vartheta_1 + \vartheta_3 \simeq 45^\circ}$
P7: $U = R_{31}(\vartheta_3)R_{12}(\vartheta_1, \phi)R_{31}^{-1}(\vartheta_2)$ $\begin{pmatrix} c_1c_3c_2 + s_3s_2e^{-i\phi} & s_1c_3 & -c_1c_3s_2 + s_3c_2e^{-i\phi} \\ -s_1c_2 & c_1 & s_1s_2 \\ -c_1s_3c_2 + c_3s_2e^{-i\phi} & -s_1s_3 & c_1s_3s_2 + c_3c_2e^{-i\phi} \end{pmatrix}$	$\vartheta_1 = (54.41_{-2.644}^{+5.748})^\circ$ $\vartheta_2 = (52.04_{-0.707}^{+1.021})^\circ$ $\vartheta_3 = (47.69_{-0.335}^{+1.885})^\circ$	$\underline{\vartheta_1 + \vartheta_3 \simeq \vartheta_2}$
P8: $U = R_{12}(\vartheta_1)R_{23}(\vartheta_2, \phi)R_{31}(\vartheta_3)$ $\begin{pmatrix} -s_1s_2s_3 + c_1c_3e^{-i\phi} & s_1c_2 & s_1s_2c_3 + c_3s_3e^{-i\phi} \\ -c_1s_2s_3 - s_1c_3e^{-i\phi} & c_1c_2 & c_1s_2c_3 - s_3s_3e^{-i\phi} \\ -c_2s_3 & -s_2 & c_2c_3 \end{pmatrix}$	$\vartheta_1 = (43.22_{-0.455}^{+2.030})^\circ$ $\vartheta_2 = (36.94_{-1.692}^{+3.691})^\circ$ $\vartheta_3 = (19.50_{-2.108}^{+3.226})^\circ$	$\vartheta_2 + \vartheta_3 = (56.44_{-3.800}^{+6.917})^\circ$ $\underline{\vartheta_2 + \vartheta_3 \simeq \vartheta_1}$
P9: $U = R_{31}(\vartheta_3)R_{12}(\vartheta_1, \phi)R_{23}(\vartheta_2)$ $\begin{pmatrix} c_1c_3 & s_1c_2c_3 - s_2s_3e^{-i\phi} & s_1s_2c_3 + c_2s_3e^{-i\phi} \\ -s_1 & c_1c_2 & c_1s_2 \\ -c_1s_3 & -s_1c_2s_3 - s_2c_3e^{-i\phi} & -s_1s_2s_3 + c_2c_3e^{-i\phi} \end{pmatrix}$	$\vartheta_1 = (30.00_{-1.455}^{+1.871})^\circ$ $\vartheta_2 = (47.76_{-0.283}^{+0.020})^\circ$ $\vartheta_3 = (17.95_{-1.916}^{+3.163})^\circ$	$\vartheta_1 + \vartheta_3 = (47.95_{-3.371}^{+5.034})^\circ$ $\underline{\vartheta_1 + \vartheta_3 \simeq \vartheta_2}$

to the tribimaximal pattern, and it has been also suggested in Ref. [14] from pure phenomenological consideration based on the T2K result.

We start with the first kind of self-complementarity relation in the P4 parametriza-

tion. With

$$R_{23}(\vartheta_2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix}, R_{12}(\vartheta_1, \phi) = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & e^{-i\phi} \end{pmatrix}, R_{31}^{-1}(\vartheta_3) = \begin{pmatrix} c_3 & 0 & -s_3 \\ 0 & 1 & 0 \\ s_3 & 0 & c_3 \end{pmatrix}, \quad (6)$$

we get

$$U = R_{23}(\vartheta_2)R_{12}(\vartheta_1, \phi)R_{31}^{-1}(\vartheta_3) \\ = \begin{pmatrix} c_1 c_3 & s_1 & -c_1 s_3 \\ -s_1 c_2 c_3 + s_2 s_3 e^{-i\phi} & c_1 c_2 & s_1 c_2 s_3 + s_2 c_3 e^{-i\phi} \\ s_1 s_2 c_3 + c_2 s_3 e^{-i\phi} & -c_1 s_2 & -s_1 s_2 s_3 + c_2 c_3 e^{-i\phi} \end{pmatrix}. \quad (7)$$

Combined with global fitting results [15], we get

$$\vartheta_1 = (33.16_{-1.014}^{+1.159})^\circ, \quad \vartheta_2 = (45.92_{-0.338}^{+0.160})^\circ, \quad \vartheta_3 = (9.98_{-2.487}^{+1.755})^\circ. \quad (8)$$

Thus we find

$$\vartheta_1 + \vartheta_3 = (43.14_{-3.501}^{+2.914})^\circ \simeq \vartheta_2 = (45.92_{-0.338}^{+0.160})^\circ \simeq 45^\circ. \quad (9)$$

As we have seen in Table 1, the self-complementarity relations are dependent on parametrizations. Though elements of the mixing matrix cannot be measured independently yet, getting the expressions of reparametrization-invariant form of the self-complementarity relations can be used to check the numerical relations to see how they work in general. This may have the potential of giving clues to find or build some underlying theories that can produce such kind of relations.

From Eq. (7), we have

$$s_1 = |U_{e2}|, \quad t_2 = \frac{|U_{\tau 2}|}{|U_{\mu 2}|}, \quad t_3 = \frac{|U_{e3}|}{|U_{e1}|}. \quad (10)$$

Substituting the trigonometric function with the moduli of the matrix elements in the following expressions,

$$\tan(\vartheta_1 + \vartheta_3) = 1, \quad \tan \vartheta_2 = 1, \quad (11)$$

we have,

$$\frac{|U_{e3}|}{|U_{e1}|} + \frac{|U_{e2}|}{\sqrt{1-|U_{e2}|^2}} = 1 - \frac{|U_{e3}||U_{e2}|}{|U_{e1}|\sqrt{1-|U_{e2}|^2}}; \quad (12)$$

$$\frac{|U_{\tau 2}|}{|U_{\mu 2}|} = 1. \quad (13)$$

With application of the unitarity relation of the first row of the PMNS matrix, namely,

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1,$$

to eliminate $|U_{e2}|$, we can get,

$$\left(\frac{|U_{e1}| - |U_{e3}|}{|U_{e1}| + |U_{e3}|} \right)^2 = \frac{1}{|U_{e1}|^2 + |U_{e3}|^2} - 1. \quad (14)$$

In similar way, we can work out the reparametrization-invariant form of other kinds of self-complementarity relations as in the P1, P9, P3 and P6 parametrizations. It is helpful to see how these expressions work with the global fitting results of the PMNS matrix, so we list the results in the third column of Table 2.

Table 2

The reparametrization-invariant forms and their verification with the numerical values from the global fit of the PMNS matrix

parametrization	self-complementarity relations	numerical values from the global fit
P4:	$\frac{1}{ U_{e1} ^2 + U_{e3} ^2} - \left(\frac{ U_{e1} - U_{e3} }{ U_{e1} + U_{e3} } \right)^2 = 1$ $\frac{ U_{\tau 2} }{ U_{\mu 2} } = 1$	$0.938^{+0.100(+0.296)}_{-0.137(-0.496)}$ $1.033^{+0.121(+0.356)}_{-0.056(-0.170)}$
P1:	$\frac{ U_{e3} (U_{e1} U_{\tau 3} + U_{e2} U_{\mu 3})}{\sqrt{1- U_{e3} ^2}(U_{e1} U_{\mu 3} - U_{e2} U_{\tau 3})} = 1$	$1.231^{+0.651(+1.807)}_{-0.373(-1.221)}$
P9:	$\frac{ U_{\mu 1} (U_{e1} U_{\mu 2} + U_{\tau 1} U_{\mu 3})}{\sqrt{1- U_{\mu 1} ^2}(U_{e1} U_{\mu 3} - U_{\tau 1} U_{\mu 2})} = 1$	$1.008^{+0.201(+0.559)}_{-0.109(-0.347)}$
P3:	$\frac{ U_{e3} (U_{\mu 1} + U_{\tau 1})}{ U_{e2} (U_{\mu 1} - U_{\tau 1})} = 1$	$0.873^{+0.263(+0.747)}_{-0.236(-0.828)}$
P6:	$\frac{ U_{\tau 1} (U_{e1} + U_{\mu 1})}{\sqrt{1- U_{\tau 1} ^2}(U_{e1} - U_{\mu 1})} = 1$	$1.132^{+0.235(+0.657)}_{-0.156(-0.514)}$

We see that the self-complementarity relations agree with the data in the 1σ (3σ) error range. Besides, we can see that for central values, the first kind of the self-complementarity relation in the P4 parametrization agrees well with the data. As it is not an exact expression, we introduce small deviation angles denoted as α , β , which satisfy

$$\tan(\vartheta_1 + \vartheta_3 + \alpha) = 1, \quad \tan(\vartheta_2 + \beta) = 1. \quad (15)$$

From the global fit [15], we get $\alpha = (1.86^{+2.914}_{-3.501})^\circ$ $\beta = (0.92^{+0.160}_{-0.338})^\circ$ in the P4 parametrization and see that α , β are indeed small. Notice that Eq. (15) is parametrization-dependent. As the matrix elements are reparametrization-invariant, it is useful to deduce the expressions of the deviations and the matrix elements. The following expressions hold exactly and α , β can be used to check the validity and generality of the self-complementarity relations,

$$\frac{1}{|U_{e1}|^2 + |U_{e3}|^2} - 1 = \left(\frac{(1 - \tan \alpha)|U_{e1}| - (1 + \tan \alpha)|U_{e3}|}{(1 + \tan \alpha)|U_{e1}| + (1 - \tan \alpha)|U_{e3}|} \right)^2; \quad (16)$$

$$|U_{\tau 2}| = \frac{1 - \tan \beta}{1 + \tan \beta} |U_{\mu 2}|. \quad (17)$$

For the self-complementarity relations in other parametrizations, we can introduce similar parameters to check the deviations of these relations from experimental global fit.

Several constant matrices are used as the zeroth-order approximation of the neutrino mixing matrix. Based on the constant matrices, theories of flavor symmetry are proposed separately. We consider four constant matrices, namely, bimaximal, hexagonal, tribimaximal and democratic mixing patterns, all of which have a vanishing θ_{13} when working out the corresponding mixing angles in the standard parametrization. Assuming the self-complementarity relation is exact, we apply it to get corrections to θ_{13} in the four constant mixing matrices and list the results in Table 3.

With the T2K results [3],

$$\theta_{13}^\nu = (9.685^{+4.698}_{-6.289})^\circ \text{ (NH)}, \quad \theta_{13}^\nu = (10.986^{+5.218}_{-6.848})^\circ \text{ (IH)}$$

and global fitting results [15],

$$\theta_{13} = 8.33^\circ \pm 1.40^\circ (\pm 4.40^\circ),$$

we find that: (1) the self-complementarity correction is compatible with the data in the error range of T2K results except for the bimaximal mixing, which

Table 3

The predictions of θ_{13} from the self-complementarity relation in constant mixing matrices

mixing matrix	mixing angles	θ_{13} from the self-complementarity relation
<u>bimaximal</u>		
$\sin^2 \theta_{12} = \frac{1}{2}$	$\theta_{12} = 45^\circ$	
$\sin^2 \theta_{23} = \frac{1}{2}$	$\theta_{23} = 45^\circ$	
$\sin^2 \theta_{13} = 0$	$\theta_{13} = 0^\circ$	$\theta_{13} = 0^\circ$
<u>hexagonal</u>		
$\sin^2 \theta_{12} = \frac{1}{4}$	$\theta_{12} = 30^\circ$	
$\sin^2 \theta_{23} = \frac{1}{2}$	$\theta_{23} = 45^\circ$	
$\sin^2 \theta_{13} = 0$	$\theta_{13} = 0^\circ$	$\theta_{13} = 15^\circ$
<u>tribimaximal</u>		
$\sin^2 \theta_{12} = \frac{1}{3}$	$\theta_{12} = 35.264^\circ$	
$\sin^2 \theta_{23} = \frac{1}{2}$	$\theta_{23} = 45^\circ$	
$\sin^2 \theta_{13} = 0$	$\theta_{13} = 0^\circ$	$\theta_{13} = 9.736^\circ$
<u>democratic</u>		
$\sin^2 \theta_{12} = \frac{1}{2}$	$\theta_{12} = 45^\circ$	
$\sin^2 \theta_{23} = \frac{2}{3}$	$\theta_{23} = 54.736^\circ$	
$\sin^2 \theta_{13} = 0$	$\theta_{13} = 0^\circ$	$\theta_{13} = 9.736^\circ$

satisfies the self-complementarity relation and needs no correction; (2) the corrections to θ_{13} in the tribimaximal mixing and democratic mixing are both close to the global fit result. There are a number of experiments to measure the neutrino mixing angle θ_{13} , with primary data already [3] or still under data taking processes [4]. These future measurements with improved precision can test the above different predictions.

Also we could start with the self-complementarity relation and seek for a constant matrix as a new mixing pattern as in Ref. [14], and the resulting matrix can provide better description to the data than other constant matrices.

In summary, we investigate the self-complementarity relations of mixing angles of lepton mixing matrix in nine different angle-phase parametrizations, work out the corresponding reparametrization invariant expressions, and make some discussions on deviations of these relations from experimental global fit. We find that the self-complementarity relations agree with the latest experimental results and can make compatible prediction when combined with some

available constant matrices. They may also lead to perspective of new mixing pattern of neutrinos. Better understanding of the self-complementarity relation may shed light on the mysterious feature of neutrino mixing and possible underlying theory behind these phenomenological regularities.

We are very grateful to Ya-juan Zheng and Zhi-zhong Xing for discussions. This work is partially supported by National Natural Science Foundation of China (Grants No. 11021092, No. 10975003, No. 11035003, and No. 11120101004) and by the Research Fund for the Doctoral Program of Higher Education (China).

Note added: There is a novel measurement of the neutrino mixing angle θ_{13} by the Daya Bay Collaboration [17], with $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ of a significance of 5.2σ , and the corresponding angle is $\theta_{13} = (8.828 \pm 0.793(\text{stat}) \pm 0.248(\text{syst}))^\circ$. The value of θ_{13} in our analysis is based on the global fit in Ref. [15], with $\theta_{13} = (8.332 \pm 1.399(\pm 4.396))^\circ$ which corresponds to $\sin^2 2\theta_{13} = 0.082 \pm 0.027(\pm 0.084)$. Therefore our analysis are compatible with the new data and we do not expect deviation from the conclusion in this work.

References

- [1] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37** (2010) 075021.
- [2] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89** (2002) 011301;
KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90** (2003) 021802;
K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90** (2003) 041801;
Super-Kamiokande Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **107** (2011) 241801;
Borexino Collaboration, G. Bellini *et al.*, Phys. Rev. Lett. **107** (2011) 141302.
- [3] T2K Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **107** (2011) 041801;
MINOS Collaboration, P. Adamson *et al.*, Phys. Rev. Lett. **107** (2011) 181802;
Double Chooz Collaboration, Y. Abe *et al.*, arXiv:hep-ex/1112.6353, (2011).
- [4] Daya-Bay Collaboration, X. Guo *et al.*, arXiv:hep-ex/0701029;
RENO Collaboration, J. K. Ahn *et al.*, arXiv:1003.1391 [hep-ex];
G. S. Davies for the NO ν A Collaboration, arXiv:1110.0112 [hep-ex];
B. J P Jones, arXiv:1110.1678 [physics.ins-det].
- [5] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531;
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
- [6] B. Pontecorvo, Sov. Phys. JETP **26** (1968) 984;
Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [7] L.L. Chau and W.Y. Keung, Phys. Rev. Lett. **53** (1984) 1802.

- [8] H. Fritzsch and Z.-Z. Xing, Phys. Rev. D **57** (1998) 594.
- [9] Y. -j. Zheng, Phys. Rev. D **81** (2010) 073009 [arXiv:1002.0919 [hep-ph]].
- [10] F. Vissani, arXiv:hep-ph/9708483;
V.D. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, Phys. Lett. B **437** (1998) 107;
A.J. Baltz, A.S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81** (1998) 5730;
I. Stancu and D.V. Ahluwalia, Phys. Lett. B **460** (1999) 431;
H. Georgi and S.L. Glashow, Phys. Rev. D **61** (2000) 097301;
N. Li and B. -Q. Ma, Phys. Lett. B **600** (2004) 248 [hep-ph/0408235].
- [11] P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B **458** (1999) 79;
P.F. Harrison, D.H. Perkins, and W.G. Scott, Phys. Lett. B **530** (2002) 167;
Z.Z. Xing, Phys. Lett. B **533** (2002) 85;
P.F. Harrison and W.G. Scott, Phys. Lett. B **535** (2002) 163;
P.F. Harrison and W.G. Scott, Phys. Lett. B **557** (2003) 76 ;
X.-G. He and A. Zee, Phys. Lett. B **560** (2003) 87;
N. Li and B. -Q. Ma, Phys. Rev. D **71** (2005) 017302 [hep-ph/0412126].
- [12] A. Y. Smirnov, arXiv:hep-ph/0402264;
H. Minakata and A.Y. Smirnov, Phys. Rev. **70** (2004) 073009;
M. Raidal, Phys. Rev. Lett. **93** (2004) 161801;
N. Li and B.-Q. Ma, Phys. Rev. D **71** (2005) 097301 [arXiv:hep-ph/0501226].
- [13] G.-N. Li, H.-H. Lin, X.-G. He, [arXiv:1112.2371 [hep-ph]].
- [14] Y.-j. Zheng and B.-Q. Ma, Eur. Phys. J. Plus **127** (2012) 7 [arXiv:1106.4040 [hep-ph]].
- [15] G. Fogli *et al.*, Phys. Rev. D **84** (2011) 053007.
- [16] Z.-z. Xing, Phys. Lett. B **696** (2011) 232.
- [17] Daya-Bay Collaboration, F.P. An *et al.*, arXiv:1203.1669 [hep-ex].